Machine Learning & Neuroscience: Performance vs. Biological Relevance in Neural Networks

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- This spike is sent down the axon to release neurotransmitters in the synapse.
- These neurotransmitters either excite or inhibit the next neuron, affecting if it spikes in voltage.

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- When voltage increases pass threshold, voltage-gated Na+ channels release Na+ inside the cell, increasing voltage.
- After some delay, voltage-gated K+ channels release K+ outside the cell, decreasing voltage.
- This pattern is known as the action potential.

Modeling the Action Potential

Leaky Integrate-and-Fire Model

$$\tau \frac{dV(t)}{dt} = -\Delta V(t) + R \cdot I(t)$$

$$\Delta V(t) = V(t) - E \qquad \text{(the driving force)}$$

where $V(t)$ is the membrane voltage, $I(t)$ is the input current, E is the resting membrane potential, $\tau > 0$ is the time constant, and R is the membrane resistance.

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\int_0^{t_0} (e^{t/\tau} y)' \, dt &= \frac{1}{\tau} \int_0^{t_0} f(t) e^{t/\tau} \, dt \\
(e^{t_0/\tau}) y(t_0) &= \frac{1}{\tau} \int_0^{t_0} f(t) e^{t/\tau} \, dt
\end{aligned}$$

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$$\boxed{y(t_0) = \frac{1}{\tau} \int_0^{t_0} f(t) e^{-(t_0 - t)/\tau} \, dt}$$

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  - For example, setting $f(t) = C$ gives

$$y(t_0) = C(1 - e^{-t_0/\tau})$$

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- We can also write this another way:

$$y(t_0) = (K * f)(t_0)$$

  where K is the exponential kernel.

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The Hodgkin-Huxley Model

$$C\frac{dV}{dt} = I - G_L(V - E) - \overline{G_{Na}}m^3h(V - E_{Na}) - \overline{G_K}n^4(V - E_K)$$

(m, n, h are each voltage and time dependent and are governed by their own set of equations)

Modeling voltages gets complicated fast!

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$$s(t) = \begin{cases} 1 & \text{a spike occurred at time } t \\ 0 & \text{otherwise} \end{cases}$$

- If we model current as exponentially decaying with each spike, we can express it as the convolution of the exponential kernel with the spike train.

The Firing Rate Equation

- This gives that current can be modeled as

$$\tau \frac{dI(t)}{dt} = -I(t) + \sum_i u_i s_i(t)$$

where u_i are the synaptic weights on the input neurons.

The Firing Rate Equation

- This gives that current can be modeled as

$$\tau \frac{dI(t)}{dt} = -I(t) + \sum_i u_i s_i(t)$$

  where u_i are the synaptic weights on the input neurons.
- If we imagine just looking at the average on a small enough time interval so that I and I' are roughly constant, we get

$$\tau \frac{dI(t)}{dt} = -I(t) + \sum_i u_i x_i(t)$$

  where x_i gives the firing rate.

The Firing Rate Equation

- We can relate the firing rate of the neuron through an $f \cdot I$-curve so that r exponentially approaches some curve $f(I)$. This gives

$$\tau_s \frac{dI(t)}{dt} = -I(t) + \sum_i u_i x_i(t)$$

$$\tau_r \frac{dr}{dt} = -r + f(I)$$

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- If we assume the firing rate changes much slower than the current changes ($\tau_s \approx 0$), we can simplify into one equation and get

$$\boxed{\tau \frac{dr}{dt} = -r + f\left(\sum_i u_i x_i(t)\right)}$$

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The Feedforward Neural Network

- Writing the firing rate equation in matrix form gives

$$\tau^T \frac{d\mathbf{r}}{dt} = -\mathbf{r} + f(U\mathbf{x})$$

where U is the synaptic weight matrix and $\mathbf{x}$ is the firing rate of the input neurons.

The Feedforward Neural Network

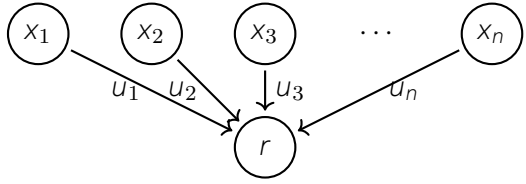
- Writing the firing rate equation in matrix form gives

$$\tau^T \frac{d\mathbf{r}}{dt} = -\mathbf{r} + f(U\mathbf{x})$$

  where U is the synaptic weight matrix and $\mathbf{x}$ is the firing rate of the input neurons.
- The steady-state of the equation is given by $\mathbf{r} = f(U\mathbf{x})$. This is a feedforward neural network with one layer.

The Feedforward Neural Network

- Each input neuron feeds into every neuron.



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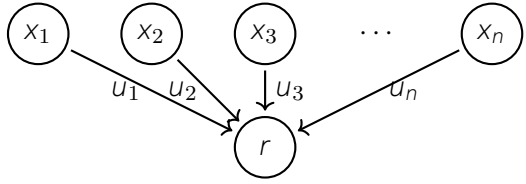
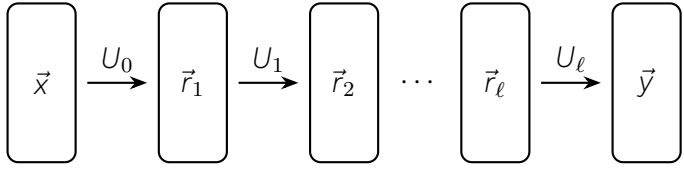


- Having multiple layers creates a deep neural network.



- An example of this in the brain may be the visual cortex.

Synaptic Plasticity

- One way to train weights is through Hebbian plasticity: *"Neurons that fire together, wire together"*. Synaptic weights are updated based on pre-synaptic and post-synpatic activity. This is an example of unsupervised learning.

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- Hinton went on to work for Google and Sutskever became a co-founder of OpenAI. In 2023, they both left their respective companies.

Is Backpropagation Biologically Relevant?

- Backpropagation relies on knowing the values of all synaptic weight matrices and iteratively multiplying by them backwards.

$$\text{error at level } k = (f_k)' \cdot W_{k+1}^T \cdot (\text{error at level } k+1)$$

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- The problem is training is *not local*. In order to change synaptic weights, neurons would have to have a separate backward pass to inform of *all* synaptic weights forward in the network.

Biologically Plausible Training

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We need on average that

$$(W_k^T\mathbf{e})^T B\mathbf{e} > 0 \qquad \text{so } \theta < 90^\circ$$
$$\mathbf{e}^T W_k B\mathbf{e} > 0$$

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  - Your brain didn't need the entire internet and a datacenter to learn how to hold a conversation!
- Computational Neuroscience also focuses on biological relevance to learn how the brain actually processes information.
- The brain is the best computer we know: learning more about the brain can help enlighten machine learning.

Recurrent Neural Networks

Add coupled weights to our inputs

$$\tau^T \frac{d\mathbf{r}}{dt} = -\mathbf{r} + f(W\mathbf{r} + U\mathbf{x})$$

Our "steady-state" is now $\mathbf{r} \leftarrow f(W\mathbf{r} + U\mathbf{x})$, giving a recurrent neural network.

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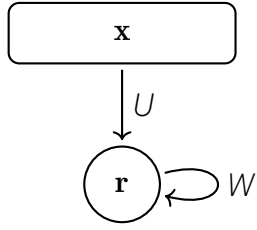

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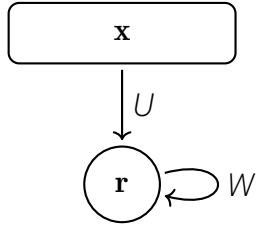

Non-linear RNNs have effectively described many behaviors such as memory (Hopfield, 1982), decision making (Wong & Wang, 2006), and vision (Ozeki et al., 2009).

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\end{aligned}$$

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$$\mathbf{a} \leftarrow D\mathbf{a} + B\mathbf{x}$$

decoupled!

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decoupled!

Essentially puts $\mathbf{r}$ into a new coordinate system, given by the eigenvectors of W.

Recurrent Neural Networks

Return back to our full differential equation:

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- When $\lambda_i < 1$, this is exponential approach.
- When $\lambda_i > 1$, this is exponential growth.

Hence, larger λ_i contribute more to the overall system. This is principal component analysis (PCA).

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- Research shows that brain activity is largely dominated by few internal signals, such as in decision making (Bondy et al., 2024).

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- Dimensionality reduction helps discover macro-level latent dynamics.
- This has been shown to be important to understanding certain complex behaviors such as motor control (Vyas et al., 2020).
- Research shows that brain activity is largely dominated by few internal signals, such as in decision making (Bondy et al., 2024).
- In other words, the brain is doing a lot with seemingly very little!
- Thus, one might shift away from model training and more towards understanding optimal latent dynamics and inputs.

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Latent Dynamics Model

Assume the following RNN architecture (Schimel et al., 2022):

$$\begin{aligned} \text{latent dynamics} \quad & \mathbf{r}_{t+1} = f_\theta(\mathbf{r}_t, \mathbf{x}_t) \\ \text{observations} \quad & \mathbf{o}_t | \mathbf{r}_t \sim p_\theta(\mathbf{o}_t | \mathbf{r}_t) \end{aligned}$$

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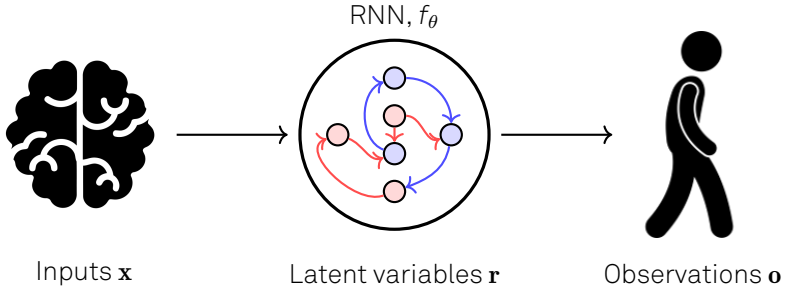
Low dimensional neuronal dynamics $\mathbf{r}$ are controlled by an RNN f_θ with inputs $\mathbf{x}$.

But actual behaviors $\mathbf{o}$ vary according to some probability function p_θ.

- $p_\theta(\mathbf{o}_t|\mathbf{r}_t)$ is chosen to be either a normal or a student t-distribution.

Latent Dynamics Model



iLQR - VAE

Estimate $q_\phi(\mathbf{x}_t|\mathbf{o}_t)$ based on *optimal inputs*. This is through an iterative Linear Quadratic Regulator (Li & Todorov, 2004).

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- Inputs are made optimal by an energetic cost given by $\log p_\theta(\mathbf{x}_t)$.

iLQR - VAE

Estimate $q_\phi(\mathbf{x}_t|\mathbf{o}_t)$ based on *optimal inputs*. This is through an iterative Linear Quadratic Regulator (Li & Todorov, 2004).

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The resulting function q is a normal distribution centered at the optimal inputs $\mathbf{x}^\star$.

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Finally, we optimize our parameters θ, ϕ on our RNN based on a cost function on our latent variables $\mathbf{r}_t$ gathered by our optimal inputs.

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The resulting algorithm aims to optimize both inputs and parameters to make the system efficient at achieving desired behavior.

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Recall:

$$\text{latent dynamics} \quad \mathbf{r}_{t+1} = f_\theta(\mathbf{r}_t, \mathbf{x}_t, t)$$
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The key is choosing the right RNN!

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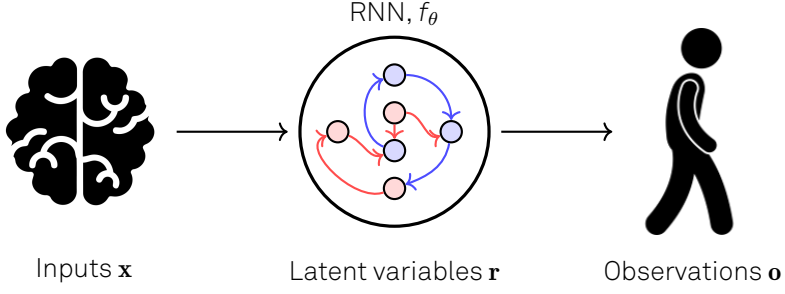
A biologically relevant RNN could better explain what the brain is actually doing, illuminating how it efficiently performs calculations.

Machine Learning & Neuroscience

- The brain works efficiently.



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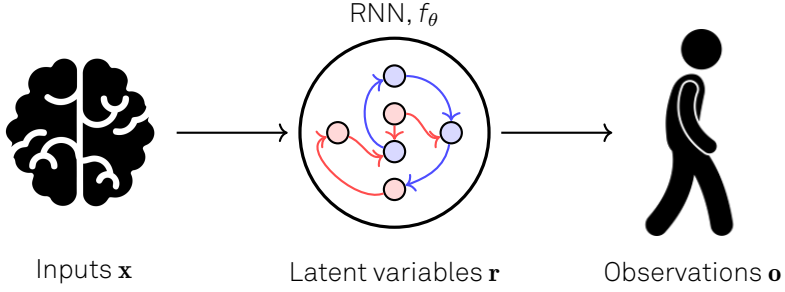

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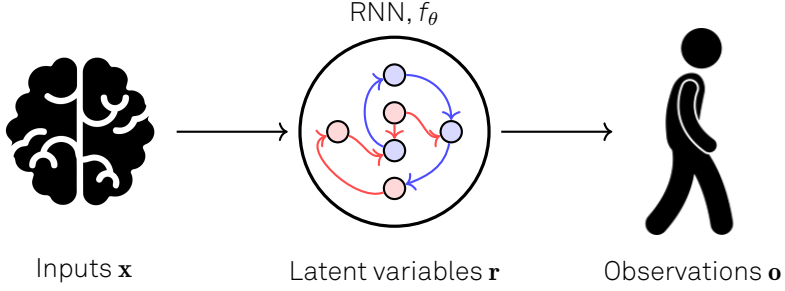

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- But if not biologically relevant, we can't get that efficiency.
- More importantly, if not biologically relevant, we can't *understand* that efficiency.